Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal, 2025, 19(6), 194–202
https://doi.org/10.12913/22998624/202891
ISSN 2299-8624, License CC-BY 4.0

Received: 2025.01.08
Accepted: 2025.04.15
Published: 2025.05.01

Monte Carlo validation of the pairwise comparisons method accuracy improvement for 3D objects

Waldemar W. Koczkodaj[1], Konrad Kułakowski[2], Meng Cheng Lau[1], Witold Pedrycz[3], Alexander Pigazzini[4], Yingli Song[1], Dominik Strzałka[5*], Tomasz Żądło[6]

[1] Laurentian University, Sudbury, 935 Ramsey Lake Rd, Sudbury, ON P3E 2C6, Canada
[2] AGH University of Krakow, al. Adama Mickiewicza 30, 30-059 Kraków, Poland
[3] University of Alberta, Edmonton, 116 St & 85 Ave, Edmonton, AB T6G 2R3, Canada
[4] Mathematical and Physical Science Foundation, Slagelse, Denmark
[5] Rzeszow University of Technology, Rzeszów, Aleja Powstańców Warszawy 12, 35-959 Rzeszów, Poland
[6] University of Economics in Katowice, ul. 1 Maja 50, 40-287 Katowice, Poland
* Corresponding author's e-mail: strzalka@prz.edu.pl

ABSTRACT

A Monte Carlo study of the pairwise comparisons method has been designed to validate the accuracy improvement by the pairwise comparisons method for 3D objects. For this, not-so-irregular objects were randomly selected. It is important to emphasize that this study focuses on testing the accuracy of the method rather than the users' skills. The users' inability to assess the volume of unrestricted random objects (e.g., a porcupine) would only deviate the results. As a side product, semi-randomly generated 3D objects can also be useful in many other research areas, such as software validation and verification, microeconomics (consumer preferences for products), computer entertainment, and even agriculture (selecting of fruits and vegetables). Further generalizations incorporating additional dimensions, as a comparison of different investment opportunities, can be useful, for example in enhancing financial decision-making processes.

Keywords: pairwise comparison, Koczkodaj inconsistency indicator, Monte Carlo method, 3D semi-random object generation, 3D printing, statistical analysis.

INTRODUCTION

The pairwise comparisons (PC or PCs) method has been in use since the 13th century when Llull documented it in [1]. The PC method is based on an old adage: When eating an elephant, take one bite at a time. This expresses a sensible approach to a challenging task by attacking it entity by entity. Entities could be objects or abstract concepts. Assigning weights to a sizable number of entities is more challenging than comparing two entities at a time. The intuitive assumption that the pairwise comparisons method is somehow superior to the direct assignment of weights was supported by the first Monte Carlo study conducted in 1996 in [2]. Random horizontal bars were displayed for users to assess their lengths in a direct way and by pairwise comparisons. In the direct way, a unit (called a brick) was used to estimate the length. Using any other unit, such as one centimeter or inch, was at that time infeasible since the character (not pixel) graphic mode was the only one available to the author. The results are compiled in Table 1.

Assessment of intangible entities (e.g., software reliability or software safety) involves not only imprecise or inexact knowledge but also inconsistency in our assessments. It is a natural approach for processing subjectivity, although objective entities can also be processed in this way.

Intuitively, it is obvious that the "two at a time approach" is better than the "everything at once" method for any set of comparisons. However, to show that the pairwise comparisons method is superior to the "by an expert's eye" common sense approach is not entirely a trivial task since there are many hurdles to overcome. One of the problems is

Table 1. Printed objects and their volumes

Number of bars	3		4		5		6	
Number of observations	139		138		129		127	
Mean error	PC	DR	PC	DR	PC	DR	PC	DR
	4.150	11.583	4.092	13.166	3.92	15.219	3.763	16.582
Standard deviation	2.866	6.195	2.671	7.572	2.507	7.918	2.458	8.905
K-S value	1.185	1.412	1.412	1.419	1.560	1.587	1.584	1.760
Critical value for $\alpha = 0.05$	0.115		0.116		0.120		0.121	

the generation of random objects. The main goal of the presented experiment is to compare the accuracy of area assessments based on the pairwise comparisons method with the direct method, which is also referred to as "by eye estimation".

THE METHOD OF PAIRWISE COMPARISONS

In the pairwise comparisons (PC) method, entities (physical objects or abstract concepts) are presented in pairs to one or more human experts. At the current stage of pairwise comparisons theory, there is no possibility of proving, or disproving, by analytical means which method is superior. This is why we need to conduct Monte Carlo studies (e.g., [3] and [4]). Stanislaw Ulam introduced the Monte Carlo method when working on the Manhattan Project. In mathematical terms, an $n \times n$ real matrix A = $[a_{ij}]$ is a pairwise comparisons (PC) matrix if $a_{ij} > 0$ and $a_{ij} = 1/a_{ji}$ for all $i,j = 1,\ldots,n$. Elements a_{ij} represent the result of ratios which are (often subjective) comparisons of the ith entity with the jth one. A PC matrix A is consistent if $a_{ij} \cdot a_{jk} = a_{ik}$ for all $i,j,k = 1,\ldots,n$. It is easy to see that a PC matrix A is consistent if and only if there exists a positive n-vector w such that $a_{ij} = w_i/w_j$, $i,j = 1,\ldots,n$. For a consistent PC matrix A, the values w_i serve as priorities or implicit weights of the importance of alternatives. More details about the problem of inconsistent assessments and definitions of inconsistency can be found, e.g., in [5, 6, 7].

CHALLENGES OF RANDOM OBJECT GENERATION FOR A MONTE CARLO STUDY

A rather straightforward 1D case (randomly generated bars) for testing the accuracy of pairwise comparisons was published in [2]. The random bar length estimation error went down from approximately 15% (by the direct method) to approximately 5% by the pairwise comparisons method. Evidently, it was not easy to find a solution to a 2D case since semi-random objects needed to be generated. This was presented in [9].

For the 2D Monte Carlo experimentation, we needed random objects. However, these random objects cannot be too complicated for their area estimation. For example, the area estimation for an image of a porcupine and sun with many rays are not easy and must be excluded as not easy to assess. On the other hand, random objects cannot be trivial to estimate. For example, the size of randomly generated rectangles or circles may be easy to assess for some tested subjects.

HEURISTIC FOR SEMI-RANDOM OBJECT GENERATION

Several heuristics were presented in [10] for the random generation of polygons and implemented as the RPG (random polygon generator) software package. Although it is possible to generate random objects, they may not be acceptable for area or volume assessments.

1D case

Table 1 shows results for the 1D case presented in [2]. Horizontal bars of random lengths were displayed on the computer monitor for assessment by subjects (undergraduate students). Each student assessed a different set of random bars.

2D case

2D shapes in [9] were generated by a heuristic method summarized in Table 2. The Gaussian

Table 2. 2D random shape generation by a heuristic

Step 1	Generate *N* random points
Step 2	Join points by thick black lines to form a closed curve
Step 3	Apply Gaussian blur with a very large radius
Step 4	Set a high threshold value λ to separate white and black

blur is obtained by applying function given by Equation 1:

$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2 + y^2}{2\sigma^2}} \quad (1)$$

where: x is the distance from the origin in the horizontal axis, y is the distance from the origin in the vertical axis, and σ is the standard deviation of the Gaussian distribution.

3D OBJECTS

Nature creates random 3D objects such as river stones, fruits, and vegetables. They have been scanned with a 3D scanner. We used the 3D printing process using a MakerBot's printer The Replicator (Dual) and Makerware software. We utilized open-source FreeCAD (https://www.freecad.org/) to create and enlarge various 3D objects. The volumes of these objects (in cubic centimeters (cm^3)) are presented in Table 3. Figure 1 shows 3D objects used in this study.

Table 3. Printed objects and their volumes

Shape ID	Name	Volume (cm^3)
S0	Cube	1.00
S1	stone_2	30.702
S2	Tomato	51.801
S3	Paprika	12.933
S4	stone_1	16.383
S5	stone_3	11.481

DATA COLLECTION AND RESULTS

We evaluated and compared the performance of the pairwise comparisons method for random 3D objects using a Monte Carlo simulation approach. Our selection criteria required participants to have a basic understanding of 3D objects, though not necessarily as experts, to ensure a broad and unbiased understanding. We recruited 33 undergraduate and graduate students from local educational institutions – this is expanded below in the text. Hence, the considered method will be presented based on a non-probability sample, which is a widely used approach with strong theoretical foundations (e.g., [11] and [12]).

Each participant completed comparisons (for instance, S1:S2, S4:S5, etc.), where they compared two random 3D objects side-by-side, assessing them based on volume. Figure 2 displays the six 3D-printed objects that are uniquely labelled with red numbers from 0 to 5. Object 0, a small, textured cube, serves as a reference for the respondents. Objects 1 through 5 correspond

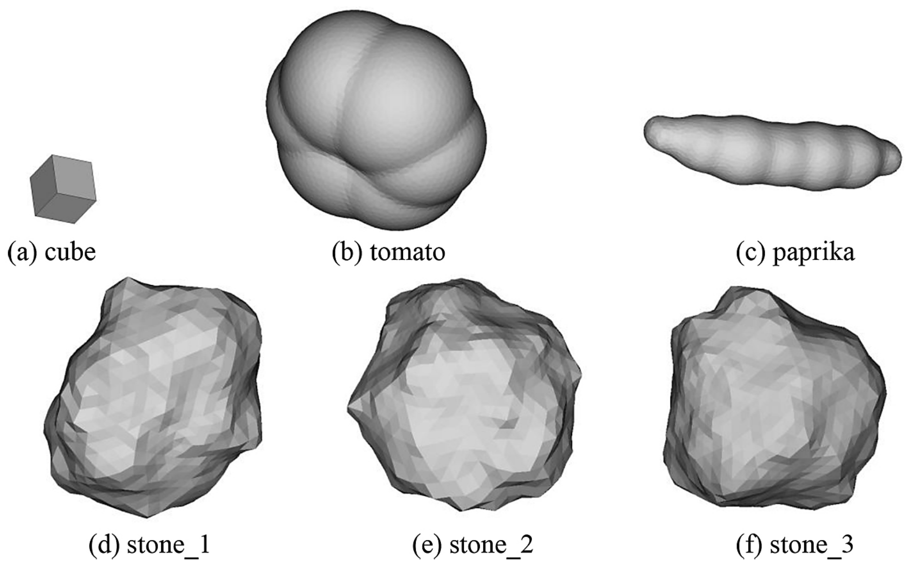

Figure 1. Six 3D objects corresponding to our study

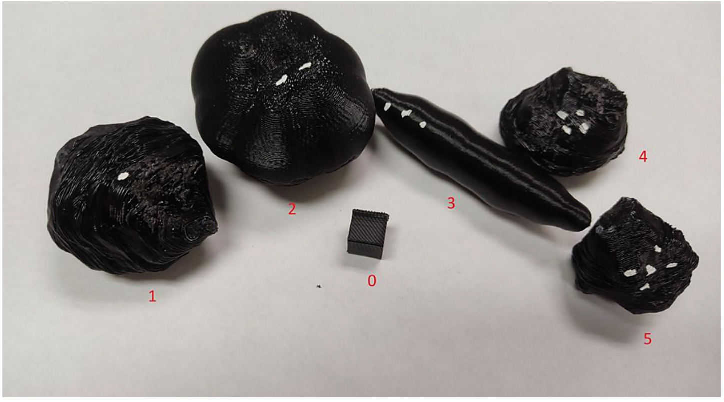

Figure 2. Labeled 3D printed objects

to objects S1 through S5 and exhibit a variety of geometries. All objects are black with white dots, which aid in analyzing their volume or surface area for the study or experiment.

Before the data collection, we briefed participants on the study's purpose without influencing their decisions and obtained their consent to ensure they understood the study's purpose and how their data would be used. We provided clear instructions on how to interact with the 3D objects, the basis for their decisions, and how to record their confidence levels. The data collection process, designed to be efficient and yield meaningful results, took approximately 2 weeks. We used a form (see Fig. 3) to collect data. For every response on a prepared form (see Fig. 3), the vector of the direct estimations (with 1 cm^3) was normalized to the sum of 1 and compared with the vector of geometric means (also normalized the same way) of the PC matrix constructed from the pairwise comparisons. Students (subjects by statistical terminology) have estimated volumes of five randomly generated 3D objects in the second part of our Monte Carlo experiment. In the first part, the same random five objects have been compared in pairs.

Let us denote the vector of the direct estimations by $D = [d_i]$, $i = 1,2,\ldots,5$. Next we denote a normalized (to the sum of 1) vector of the direct estimations $w = [w_i]$, where $w_i = [d_i/\Sigma d_i]$, and denote the PC matrix $C_{ij} = [d_i/d_j]$.

The geometric mean of a row is:

$$M_i \left(\prod_{j=1}^{n} C_{ij}\right)^{\frac{1}{n}} \tag{2}$$

Shapes are marked with dots from one dot to five dots assigned randomly (do not assume any order of them!) In mathematics, a **ratio** shows how many times one number contains another giving the value of their quotient e.g. a:b, a to b, or a/b.

*For example: If the size of **A is twice the size of B**, the ratio of A to B can be written as:*
A : B = 2 or 1.75 or 1 1/2 or 3/4 .

1. Estimate the ratio between two shape volumes (a decimal point with one or two digits may be used) and your confidence level from 1 to 3 where 3 is the highest.

Shapes	**Ratio**	**Confidence Level**	**Shapes**	**Ratio**	**Confidence Level**
S1 : S2			S2 : S4		
S1 : S3			S2 : S5		
S1 : S4			S3 : S4		
S1 : S5			S3 : S5		
S2 : S3			S4 : S5		

2. Estimate the volume of each shape S in cm^3 (a 1 cm^3 cube is provided, and it is the only shape without dots) and your confidence level (CL) from 1 to 3 where 3 is the highest.

S1 =_______ cm^3, S2 =_______ cm^3, S3=_______cm^3, S4=_______cm^3, S5=_______cm^3

CL =_______ CL =_______ CL =_______ CL =_______ CL =_______

Gender: Male/Female/Other/Prefer not to say **Age**: ______ **Program**: BSc / MSc

Time Taken: ______

Figure 3. Data collection form

where: n is the PC matrix size and $i = 1,2,...,5$ denotes the chosen row.

The geometric mean vector is: $M_v = [M_i]$. Then, we can rewrite the geometric mean of a row as:

$$M_i \left(\prod_{j=1}^{n} d_i/d_j\right)^{\frac{1}{n}} \quad (3)$$

If we normalize the geometric means of each row, we obtain:

$$M_{w_i} = M_i/\Sigma M_i \quad (4)$$

or equivalently:

$$M_{w_i} = \left(\frac{d_i^5}{d_1 \cdot d_2 \cdot d_3 \cdot d_4 \cdot d_5}\right)^{\frac{1}{5}} / \Sigma \left(\frac{d_i^5}{d_1 \cdot d_2 \cdot d_3 \cdot d_4 \cdot d_5}\right)^{\frac{1}{5}},$$

then:

$$M_{w_i} = \frac{d_i}{(d_1 \cdot d_2 \cdot d_3 \cdot d_4 \cdot d_5)^{1/5}} / \frac{\sum d_i}{(d_1 \cdot d_2 \cdot d_3 \cdot d_4 \cdot d_5)^{1/5}},$$

or:

$$M_{w_i} = \frac{d_i}{(d_1 \cdot d_2 \cdot d_3 \cdot d_4 \cdot d_5)^{1/5}} \cdot \frac{(d_1 \cdot d_2 \cdot d_3 \cdot d_4 \cdot d_5)^{1/5}}{\sum d_i} =$$

$$= d_i/\Sigma d_i,$$

i.e., $w_i = M_{wi}$.

Q.E.D.

Based on the above mathematical formulas, the main algorithm (see Fig. 4) is: the difference between two vectors x and y is the Euclidean distance

$$\delta = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{1/2} \quad (5)$$

where: n is the size of vectors (number of observations).

Five numbers (creating one vector D) estimated by each student and five exact volumes (creating vector E) should be normalized and the Euclidean distance δ between D and E is computed (see Fig. 5). For each student, we create one PC matrix and geometric means of rows normalized to the sum of 1.

The sample size

The volumes of our five assessed objects are known, real values; hence, their standard deviation and the mean do not need to be estimated but can be computed [13, 14, 15]. For this reason, there is no need to estimate the minimum sample size to achieve the assumed level of accuracy since it can also be computed by what follows. The natural choice for defining the required minimum sample size is to set it as the sample size that produces a relative standard error of the mean estimator given by (e.g. [16] p. 397 and [17] p. 473):

$$\frac{\sigma}{\sqrt{n}} \mu^{-1} \quad (6)$$

lower than the assumed value of ϵ (i.e. $\sigma\mu^{-1}n^{-0.5} < \epsilon$). Consequently, the formula is given by: $n > \sigma^2\mu^{-2}\epsilon^{-2}$ where σ^2 is the population variance, and μ is the population mean, both known in our case.

We assume that the relative standard error cannot be higher than $\epsilon = 2.5\%$, which is smaller than the commonly accepted by researchers values of 3% and 5%. The resulting value is $n > 24.24$, which leads to the minimum sample size of $n = 25$. The small size sample is the result of a small variance σ^2. It is so since even one large ratio between two objects (hence a PC matrix element) can deviate computation. The large value in one pair disqualifies other pairwise comparisons to influence the final results significantly. The final sample size in our case was 33, as no additional effort was needed compared to the minimum sample size.

STATISTICAL ANALYSIS OF ESTIMATION ERRORS

Volumes of objects (the real values) are known. They will be denoted by θ_d, where $d = 1,2,...,D$ (here $D = 5$). These values are estimated using two methods, namely, the PC method and the direct method. Let the estimate be denoted by $\widehat{\Theta}_d^i$,

The main algorithm
Input data
part 1 = pairwise comparisons (the upper triangle of a PC matrix) M
part 2 = D vector of direct estimations
E = vector of the exact volumes

1. Compute vector w as a normalized (to the sum of 1) vector of D
2. $w_i = [d_i/\Sigma d_i]$
3. Compute a normalized vector GM of geometric means of M
4. Compute the difference between vector w and vector E
5. Compute the difference between vector GM and vector E

Figure 4. Algorithm for data analysis

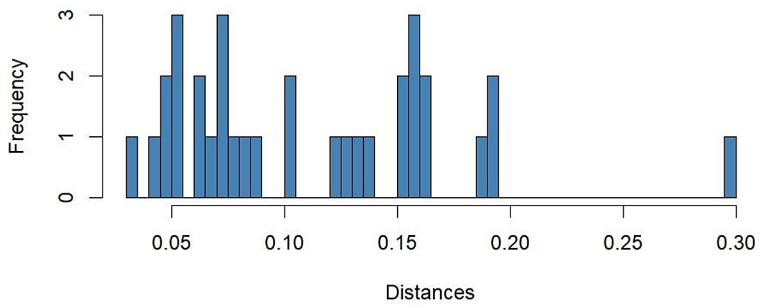

Figure 5. Euclidean distances between Vector D and Vector E

where $i = 1,2\ldots, n$ (here $n = 33$ is the number of subjects/students). The issue, from the technical point of view, is similar to two classic problems. The first one is known in classic econometrics as the ex-post assessment of the prediction accuracy or, in machine learning, as the prediction accuracy assessment via k-fold cross-validation (see [18]). The second one involves measuring accuracy in the design-based approach in survey sampling via Monte Carlo simulation studies (e.g., [19]). The main difficulty in the analysis is that the problem is not univariate. We aim to compare the accuracy of the estimation of five parameters ($D = 5$ volumes of objects), not just a single parameter.

Firstly, let us propose the following measure of accuracy estimation of D parameters, which will be called the Average Mean Squared Error. It is inspired by the average test MSE obtained via k-fold cross-validation for machine learning modeling. It is given by (compare [18] p. 181):

$$AvMSE(\widehat{\Theta}_1, \ldots, \widehat{\Theta}_d, \ldots, \widehat{\Theta}_D) = (n \times D)^{-1} \sum_{d=1}^{D} \sum_{i=1}^{n} \left(\widehat{\Theta}_d^i - \Theta_d\right)^2 = (n \times D)^{-1} \sum_{d=1}^{D} \sum_{i=1}^{n} \left(u_d^i\right)^2 \quad (7)$$

where: $u_d^i = \widehat{\Theta}_d^i - \Theta_d$ are estimation errors. The value of the measure computed for normalized errors of the direct estimation method is approximately 0.00319. For the PC method, it is approximately 0.00248. This indicates that the PC method is 22.3% more accurate than the direct method.

While measures based on the average of squared errors are widely used in statistics, it should be noted that squared errors tend to be strongly and positively skewed. In such cases, simply computing the average as the central tendency measure may not be sufficient or appropriate. Hence, we would like to propose easily interpreted measures allowing for estimation accuracy comparisons. To assess the accuracy of multivariate estimation, we will adopt measures proposed by [20] for a multivariate prediction problem.

Firstly, we propose to use the quantile of mixture of absolute estimation error (QMAEE) of order p given by (compare [17] p. 415):

$$QMAEE_p(\widehat{\Theta}_1, \ldots, \widehat{\Theta}_{\mathrm{d}}, \ldots, \widehat{\Theta}_{\mathrm{D}}) = q_p(|u_1^1|, \ldots, |u_d^i|, \ldots, |u_D^i|) \quad (8)$$

For the considered case, it is a quantile of the mixture with equal weights of errors for the D-variate estimation problem. For the PC estimation method, it is a quantile of the assumed order computed based on all absolute errors presented in five boxplots on the left-hand side of Figure 6, and for the direct method based on 5 boxplots on the right part of the same figure. It is proposed to utilize the quantiles of order $p = 0.5$ to evaluate the central tendency of estimation accuracy. However, higher-order quantiles (e.g., $p = 0.9$) should also be considered if the researcher seeks to not only assess the average accuracy (represented by the median), but also identify possible large errors within the upper-tail of the absolute estimation error distribution. This approach enables a comprehensive assessment of the estimation accuracy, allowing for a more nuanced understanding of the underlying data. As such, researchers are advised to consider the application of higher-order quantiles where appropriate, to effectively identify and address potential errors that may have a significant impact on the accuracy of the estimates. For the PC estimation method, we obtained $QMAEE_{0.5}(\widehat{\Theta}_1, \ldots, \widehat{\Theta}_5) = 0.0244$, signifying that at least 50% of all normalized estimation errors are smaller or equal to 0.0244, while $QMAEE_{0.9}(\widehat{\Theta}_1, \dots, \widehat{\Theta}_5) = 0.0774$ indicating that at least 90% of all normalized estimation errors are smaller or equal to 0.0774.

The direct estimation method yielded $QMAEE_{0.5}(\widehat{\Theta}_1, \dots, \widehat{\Theta}_5) = 0.0313$ and $QMAEE_{0.9}(\widehat{\Theta}_1, \dots, \widehat{\Theta}_5) = 0.0898$. These values exceed the corresponding values obtained based on the PC method by 27.9% and 15.9%, respectively, showing that the direct method is less accurate if the accuracy is compared based on *QMAEE*. Secondly, if the variability of estimation errors for different estimated characteristics is high (e.g., θ_d, where $d = 1,2,\dots,D$, are of different orders of magnitude), or if we would like to obtain results even simpler to understand and interpret, we propose to replace the estimation errors in Equations 7 and 8 by relative estimation errors given by $r_d^i = \frac{\widehat{\Theta}_d^i - \Theta_d}{\Theta_d}$. The Relative Quantile of Mixture of Absolute Estimation Error (RQMAEE) of order p is given by (compare [20] p. 415):

$$RQMAEE_p(\widehat{\Theta}_1, \dots, \widehat{\Theta}_d, \dots, \widehat{\Theta}_D) = \\ = q_p(|r_1^1|, \dots, |r_d^i|, \dots, |r_D^i|) \quad (8)$$

Table 4. Comparison of accuracy of estimation methods based on various measures

Measure	PC method	Direct method	Gain in accuracy
av. Euclidean dist.	0.0958	0.1119	14.4%
AvMSE	0.0025	0.0032	23.3%
$QMAEE_{0.5}$ $QMAEE_{0.9}$	0.0244 0.0774	0.0313 0.0898	21.8% 13.7%
$RQMAEE_{0.5}$ $RQMAEE_{0.9}$	14.68% 48.48%	19.43% 56.33%	24.4% 13.9%

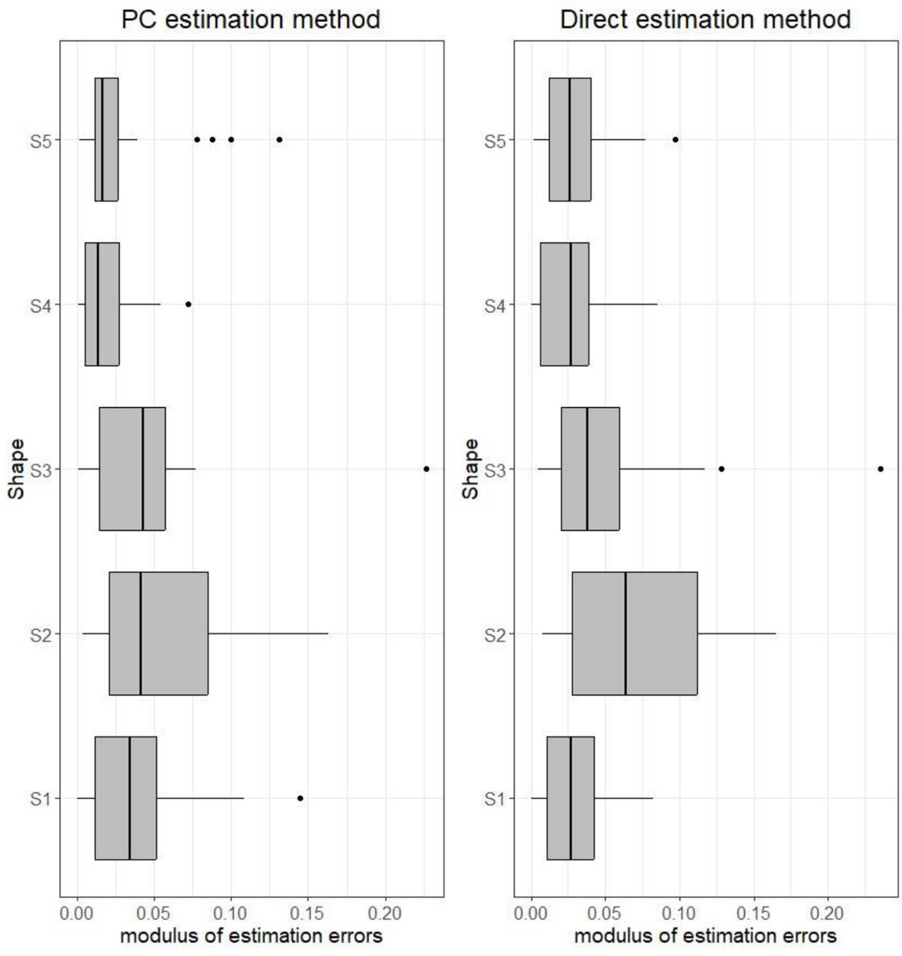

Figure 6. Modulus of errors for PC and direct estimation methods

The median of the modulus of relative normalized estimation errors was found to be 14.7% for the PC method and 19.4% for the direct method. Additionally, the quantile of order 0.9 of the modulus of relative normalized estimation errors, which indicates a pessimistic accuracy scenario, was found to be 48.5% for the PC method and 56.3% for the direct method. Thus, it can be concluded that, while the PC method provides a higher accuracy level compared to the direct method for the same input data, the observed accuracy for both methods is not particularly high.

Table 4 shows comparisons of the accuracy of estimation methods. A smaller value of the accuracy measure indicates better accuracy. The observed gains in accuracy depend on the accuracy measures used and range from 13.7% to 24.4%. Let us consider two examples to illustrate the accuracy. A 14.4% gain in accuracy using the average Euclidean distance means that the average Euclidean distance between the estimates and known true values calculated for the PC method is 14.4% smaller than for the direct method. Similarly, a 24.4% gain in accuracy based on $RQMAEE_{0.5}$ indicates that the median of the modulus of relative estimation errors (i.e., $RQMAEE_{0.5}$) calculated for the PC method is 24.4% smaller than for the direct method.

To the best of our knowledge (based on intensive searches of Web of Knowledge, Scopus, and Google), the presented results for 3D objects have never been obtained for pairwise comparisons before. As such, they are subject to improvements by subsequent research efforts.

CONCLUSIONS

The presented Monte Carlo study for the pairwise comparisons method for 3D semi-random objects has confirmed the improvement of accuracy. The gain in accuracy is 14.4% for the average Euclidean distance and 24.4% for $RQMAEE_{0.5}$. Such a gain should be considered as essential. We have also learned that generating semi-random 3D objects is not a trivial task. Printing most of the examined objects has taken hours on a printer of (more or less) a standard level of quality.

Our approach can be also applied to improve comparison of various metaheuristics as well as for a more accurate analysis of expected time efficiency of dynamic programming. In a follow-up study, more randomness will be facilitated by object selection from an image table.

Acknowledgements

We would like to thank the volunteers who assisted with data collection. Ian Corkill provided editorial improvements to our text. Financial support – Konrad Kułakowski has been supported by The National Science Centre, Poland, project no. 2021/41/B/HS4/03475. Waldemar W. Koczkodaj has been (in part) supported by The Human Capital Initiative (HCI). Tomasz Żądło's work has been cofinanced by the Minister of Science under the Regional Initiative of Excellence programme.

REFERENCES

1. Llull R., Artificium electionis personarum, Biblioteca Apostolica Vaticana, Cod. Vat. lat. 9332, f. 11r-12v) (before 1283).
2. Koczkodaj W.W., Statistically accurate evidence of improved error rate by pairwise comparisons, Perceptual and Motor Skills 1996, 82(1), 43–48. https://doi.org/10.2466/pms.1996.82.1.43
3. N. Metropolis, S. Ulam, The Monte Carlo method, Journal of the American Statistical Association 1949, 44(247), 335–341. https://doi.org/10.1080/01621459.1949.10483310
4. A. Barbu, S. C. Zhu, Monte Carlo methods, Springer, Singapore, 2020. https://doi.org/10.1007/978-981-13-2971-5
5. Koczkodaj W.W., Szybowski J. The limit of inconsistency reduction in Pairwise comparisons, International Journal of Applied Mathematics and Computer Science, 2016, 26(3), 721–729. https://doi.org/10.1515/amcs-2016-0050
6. Koczkodaj W.W., Szybowski J., Wajch E. Inconsistency indicator maps on groups for pairwise comparisons, International Journal of Approximate Reasoning, 2016, 69, 81–90. http://dx.doi.org/10.1016/j.ijar.2015.11.007
7. Smarzewski R., Rutka P. Consistent projections and indicators in pairwise comparisons, International Journal of Approximate Reasoning, 2020, 124, 123–132, https://doi.org/10.1016/j.ijar.2020.06.001
8. Amenta P., Lucadamo A., Marcarelli G. On the transitivity and consistency approximated thresholds of some consistency indices for pairwise comparison matrices, Information Sciences, vol. 2020, 507, 274–287. https://doi.org/10.1016/j.ins.2019.08.042
9. Koczkodaj W.W. Testing the accuracy enhancement of pairwise comparisons by a Monte Carlo

experiment, Journal of Statistical Planning and Inference 1998, 69(1), 21–31. https://doi.org/10.1016/S0378-3758(97)00131-6
10. Auer T., Held M. Heuristics for the generation of random polygons, in: Fiala F., Kranakis E., Sack J.R. (Eds.) Proc. 8th Canad. Conf. Computat. Geometry, Carleton University Press, Ottawa, Canada, 1996, 38–44. https://doi.org/10.1515/9780773591134-009
11. Valliant R., Dorfman A.H., Royall R.M. Finite population sampling and inference: a prediction approach, Wiley, New York, 2000.
12. Wu C. Statistical inference with non-probability survey samples. Survey Methodology, Statistics Canada 2022, 48(2), 283–312. http://www.statcan.gc.ca/pub/12-001-x/2022002/article/00002-eng.htm
13. Ryan T. P. Sample size determination and power. John Wiley & Sons, New Jersey, 2013.
14. Kieser M. Methods and applications of sample size calculation and recalculation in clinical trials. Springer, Cham, 2020.
15. Verma J.P., Verma P. Determining sample size and power in research studies. Springer, Singapore, 2020.
16. Wackerly D. D., Mendenhall W., and Scheaffer R. L. Mathematical statistics with applications. Vol. 7., Thomson Brooks/Cole, Belmont, 2008.
17. Fisz M. Probability Theory and Mathematical Statistics, 3rd edition, New York, Wiley, 1980.
18. James G., Witten D., Hastie T., Tibshirani R. An introduction to statistical learning with applications in R, Springer, New York, 2021. https://doi.org/10.1007/978-1-0716-1418-1
19. Antal E., Tillo Y. A direct bootstrap method for complex sampling designs from a finite population, Journal of the American Statistical Association 2011, 106(494), 534–543. https://doi.org/10.1198/jasa.2011.tm09767
20. Żądło T. On asymmetry of prediction errors in small area estimation, Statistics in Transition New Series 2017, 18(3), 413–432. https://doi.org/10.59170/stattrans-2017-021
21. Rudek R. A generic optimization framework for scheduling problems under machine deterioration and maintenance activities, Computers & Industrial Engineering, 2022, 174, 108800.1–22.
22. Rudek R. Scheduling on parallel processors with varying processing times, Computers & Operations Research, 2017, 81, 90–101. https://doi.org/10.1016/j.cor.2016.12.007
23. Koczkodaj W.W., Mazurek M., Strzalka D., Wolny-Dominiak A., Woodbury-Smith M. Electronic Health Record Breaches as Social Indicators. Soc Indic Res 2019, 141, 861–871. https://doi.org/10.1007/s11205-018-1837-z
24. Dymora P., Mazurek M. and Łyczko K. The effectiveness analysis of selected IT tools for predictions of the Covid-19 pandemic, 2022 17th Conference on Computer Science and Intelligence Systems (FedCSIS), Sofia, Bulgaria, 2022, 583–586. https://doi.org/10.15439/2022F65